УДК 624.044 : 539.376

ANALYSIS OF STRUCTURAL EFFECTS OF TIME-DEPENDENT BEHAVIOUR OF CONCRETE: AN INTERNATIONALLY HARMONIZED FORMAT

Mario Alberto CHIORINO, Prof.

Modern concrete structures, realized through complex sequential construction techniques and/or characterized by significant non-homogeneities, are in general very sensitive to the effects of time-dependent behaviour of concrete (creep and shrinkage).

Guidelines for the evaluation of these effects were developed in the last decades by international pre-standard and standard institutions on the basis of a common, although progressively evolving, scientific background, and of a substantially worldwide harmonized format.

The author discusses the development, with his large personal involvement, of this favourable scenario, evidencing areas of well established consensus and open problems.

In what concerns more specifically the effects of creep, it is commonly accepted that a reliable analysis of the structural response in service conditions may be performed on the basis of the theory of ageing linear viscoelasticity, first established by Italian mathematician Volterra at the dawn of 20th century.

The paper discusses the computational implications of this approach with reference on the one hand to the adoption of realistic advanced models for the prediction of the creep behaviour of concrete, and, on the other hand, to the complexity and sequential character of the constructions, and illustrates current updated guidelines developed at the international level for the evaluation of the effects of creep, both in the conceptual and preliminary design stages and in the subsequent detailed construction-stage and long-term reliability analyses of complex and sequential structures . These guidelines are intended to deal also with other phenomena, which are responsible of causing deviations from aging linear viscoelasticity, like tensile cracking, cyclic creep, and stress relaxation in prestressing tendons at variable strain, as well as the effects of humidity and temperature variations.

The paper must be intended also as a homage to the memory of CEB (Comité Euro-International du Béton, Euro-International Committee for Concrete) Honorary Member and member of the Academy of Construction and Architecture of the USSR Alexei A. Gvozdev, for long-time head of the laboratory of reinforced concrete of NIIZhB, the Institute for Concrete and Reinforced Concrete now named after him, for his crucial role in encouraging and assisting the author in the initial steps of transporting into CEB and FIP (Fédération Internationale de la Précontrainte, International Federation for Prestressing) ambient the fundaments of this new advanced format for creep analysis, to which the school of Soviet scientists and Gvozdev himself had given a fundamental contribution.

Key words:

Concrete, shrinkage, creep, aging linear viscoelasticity, structural effects, sequential constructions, conceptual design, construction-stage and long-term reliability analyses, codes and recommendations, A.A. Gvozdev school.

1. Introduction

Modern concrete structures, realized through complex sequential construction techniques and/or characterized by significant non-homogeneities, are in general very sensitive to the effects of the delayed deformations exhibited by the time-dependent behaviour of concrete (creep and shrinkage). Typical examples are large span cantilever and cable-stayed bridges, concrete arches prestressed by jacking, composite steel-and-concrete structures, concrete or steel-and-concrete high-rise and supertall buildings. Some of these examples represent extreme recent applications of structural concrete.

Hence, a realistic evaluation of the effects of time-dependent behaviour of concrete on the structural reliability during the construction stage and along the time represents an important aspect of the design and performance assessment process.

This requires, on the one hand, the definition of reliable models for the prediction of creep and shrinkage phenomena (a material properties problem) and, on the other hand, the development of reliable computational methods for the evaluation of the structural effects of these phenomena, with an accuracy degree appropriate to the specific case (a structural analysis problem). These two problems are interrelated, but are frequently considered independent in the design practice and dealt with separately.

The first of these problems, with particular regards, in what concerns creep, to the prediction of long-term trends and values (for a duration up to 100 years or more, corresponding to the specified service-life for reliability assessments of structures of greater importance [25]), is characterized by a pending debate within the international scientific and technical community. A concise state-of-the-art is presented here at Section 3.

The present paper deals especially with the second of the two problems, namely the evaluation of the response of the structure along the time to creep and shrinkage strains, with particular emphasis to the effects of creep.

While the early computational approaches for this problem were based on too crude simplifications, adequate criteria were gradually developed in the sequel for adapting to the need of design and reliability assessments the results of research made progressively available in the fields of structural mechanics and theoretical rheology. This transfer of knowledge from the scientific domain to the domain of technical guidance documents and recommendations was coordinated and promoted by international pre-standard and standard institutions in the frame of a worldwide extensively harmonized scenario. The author was significantly involved in this process.

As a result of this progression, it can be stated that the evaluation of the structural response to the time-dependent behaviour of concrete benefits today of a set of internationally agreed fundaments and basic rules of application.

After briefly retracing the historical development in the last decades of this favourable scenario, the paper presents an overview of current state of code recommendations and guidance documents in this specific field at the international level, evidencing the areas of well established consensus and a few remaining open items.

2. Gradual development of an harmonized format for codes and recommendations worldwide

In the decade 1960-70, the attention of pre-standard and standard institutions concentrated essentially on the first of the two problems evidenced in the introduction, i.e. the prediction of time-dependent behaviour of concrete: first CEB Comité Européen du Béton (European Committee for Concrete) and then ACI American Concrete Institute implemented their Recommendations and guidance documents with specific prediction models for creep and shrinkage strains of concrete.

A few years later, CEB (renamed Comité Euro-International du Béton, Euro-International Committee for Concrete in 1976) considered that the moment was ripe to establish appropriate guidelines for the second problem, i.e. the evaluation of the response of the structures to these delayed strains. After the inclusion of a preliminary concise section in CEB Model Code 1978 [18], CEB decided to establish, under the coordination of the author, a specific Design Manual [22] entirely devoted to this subject to be considered as an extended guide for both code makers and practicing engineers. The publication in the early eighties of this CEB Manual, which was based on a compendium of the most advanced research at the time, favoured the establishment of a consensus at the international level on a common scientific background and harmonized format for code recommendations and technical guidance documents in the field of time-dependent analysis of concrete structures.

Although progressively evolving to incorporate further research advances and opportunities offered by enhanced modern computational techniques, this same scientific background and harmonized format have represented through more than two decades, and still represent, the term of reference for most of the documents, specifically devoted to this subject, or containing guidance criteria for this particular aspect of reliability assessment, developed by international pre-standard and standard institutions.

While specific sections were edited under the coordination of the author within CEB-FIP Model Code 1990 [19] and within *fib* Structural Concrete Textbook [23], a précis was autonomously incorporated in 2005 (with some imprecisions to be revised soon) by CEN (the European Committee for Standardization) in Appendix KK “Structural effects of time dependent behaviour of concrete” of Eurocode 2: Design of concrete structures - Part 2: Concrete bridges [24].

More recently, this updated knowledge has formed the basis for Section 7.2.4 “Analysis of structural effects of time-dependent behaviour of concrete”, edited by the author, of *fib* Model Code 2010 published in 2013 [25], and of the current extended draft of ACI 209.3R-XX guidance document “Analysis of Creep and Shrinkage Effects in Concrete Structures”, edited with the coordination of the author and approved in 2013 within ACI Committee 209 Creep and Shrinkage in Concrete, currently under final revision by ACI TAC Technical Activity Committee to become an official ACI Guide [2].

In the process of revision and updating of Eurocode 2 currently under way, the present text of Appendix KK to Part 2 [24] shall be properly revised and will most probably be incorporated in the main body of the Code as a guidance rule for the evaluation of the effects of time-dependent behaviour of concrete in any kind of structures, paying attention to the advances of the debate and guidance criteria developed within ACI and in the pre-standard ambient of *fib*.

In the scenario of the establishment of the planned new *fib* Model Code 2020, in what concerns the aspect of prediction models for creep and shrinkage attention will be paid to the new advances in implementation and analyses of data banks, as well as to the need to provide, at the best of current knowledge, prediction models for new types of concrete, as ultra-high performance concretes and concretes with supplementary cementitious materials. For the problem of the analysis of structural effects of time-dependent behaviour of concrete the basic guidance criteria of *fib* Model Code 2010 will be essentially maintained, while paying attention to the progresses in the provision of accurate and efficient numerical methods based on the rate-type approaches for dealing with structures highly sensitive to the time dependent behaviour of concrete, with particular regard to complex sequential and spatial structures (see Sections 3 and 4.2.1 in the following).

3. Creep and shrinkage prediction models

As stated in the introduction, the problem of evaluation of the response of the structure along the time to the time-dependent behaviour of concrete, which is discussed in this paper, cannot be fully separated from the former problem concerning the establishment of reliable prediction models for this behaviour, with particular regards to creep strains. Therefore, before discussing the formats for the evaluation of the structural response, mention is briefly made here to the influence that the forms adopted for the constitutive law characterizing the creep prediction model being adopted may exert on the computational tools adopted for this evaluation.

A state-of-the-art on modern creep prediction models at the time of its publication is presented in ACI 209.2R-08 Guide edited in 2008 [1]. The subject is further debated e.g. in [21]. ACI 209.2R-08 Guide outlines the problems and limitations in developing prediction models and illustrates and compares the prediction capabilities of four models: ACI 209R-92, Bazant-Baweja B3, CEB MC90-99 and GL2000, representing, at the date of publication of the Guide, the most widely diffused prediction models at the international level among those developed in the past two to three decades. At this respect, it must be noted that CEB MC90-99 model was recently substituted by the revised fib MC 2010 model (see Section 5.1.9.4 in [25] and the related background document), while Bazant-Baweja B3 model has been revised and substituted by Model B4, as indicated in the sequel.

Persisting divergences and uncertainties are discussed in detail in [1] and [21]; for an overview of the most advanced stages of the inherent debate, reference can be made to these documents and to the included cited literature (see e.g. here [9-11] and the background document to [25]). Main divergences concern the criteria for predicting long-term trends and values for creep, due to the persisting contrast between the need to perform reliability assessments of structures of greater importance for duration up to 100 years or more [25] and the current availability of an internationally agreed database of laboratory tests still being biased toward short creep durations (less than one decade for the majority of data, with few exceptions), in the pending lack of exhaustive material science knowledge to solve the problem through physically based constitutive approaches. Mention must be made of recent discussions concerning the possibility to refer for long-term calibration of creep models, in addition to the database of laboratory tests, to a recently established database of long-term excessive deflections of long- span bridges [8], and the inherent advantages (observations extended on a multi-decade scale) and difficulties (data related to structures characterized by complex multi-faceted mechanical behaviour along the time, and not to the material itself, thus implying delicate inverse analysis approaches). Based on these last considerations a new creep prediction model named Model B4 has recently been proposed by Bazant and his research group. See [14,15,38-40] and referenced literature therein.

A general recommendation emerging from the debate underlines the advantages of the execution in each specific case of short-time tests in drastically reducing the uncertainties of prediction models [see e.g. 9, 21].

In what concerns more specifically the relation between the formats adopted for the creep prediction models and the subsequent problem of the evaluation of the response of the structures along the time discussed in the sequel in Section 4, it must be pointed out that in most of currently recommended prediction models (like e.g. the four models incorporated in the ACI 209.2R-08 Guide [1]) the stress induced strains along the time (initial plus creep strains) are represented through the compliance function $J(t, t')$, defined within the assumption of ageing linear viscoelasticy as the strain response at time t to a sustained unit stress applied at time t'. As well established within the theory of ageing linear viscoelasticity, and detailed in Section 4.1, the association of the principle of superposition, which is implied by this theory, to model the age-dependent strain responses to stress increments applied at different times t', gives the stress-strain constitutive relation for concrete creep in the form of a Volterra integral equation characterized by the presence of an history integral.

Therefore, when it is the history of the stresses in the structure to be determined in the problem under consideration, an appropriate numerical algorithm for the solution of integral equations is required, in the lack of available analytical solutions for current forms of the compliance function $J(t, t')$ (see Section 4.2). In all the cases where a refined structural analysis is requested both in the time and in the space, as e.g. in the case of creep-sensitive complex heterogeneous and sequential structures, this algorithm must be associated to the numerical computational procedure adopted for the analysis of the structure in the space, as e.g. 3-D finite element analysis.

On the other hand, as discussed in Section 4.2.1, computation of structural effects due to creep along the time can be made more efficient approximating the integral-type constitutive law with a rate-type relation, which is also advantageous for the association with finite element method. The rate-type approach is essential also to deal with the phenomena indicated in Section 4.1 causing deviations from the principle of superposition in time (i.e. from the solutions based on aging linear viscoelasticity), which may affect the response of the structure and must be taken into account in refined analyses [9,37]. For this alternative computational option, the compliance function issuing from the prediction model being considered must be converted to an equivalent rate-type creep law. At this respect, it is worth noting that, if the four models currently incorporated in the ACI 209.2R-08 Guide [1] are considered, in the only case of Model B3 [7] the conversion to a rate-type creep law of the basic creep component of the compliance is particularly easy, as this component is already originally defined in this model by its time rate on the basis of the solidification theory [12,13].

4. Time-dependent structural analysis

4.1 Aging linear viscoelasticity

As anticipated in the preceding Section, in what concerns the effects of creep, it is commonly accepted that a reliable analysis of the structural response in service conditions (i.e. for stresses less than about 0.4 of the concrete strength) may be performed on the basis of the theory of ageing linear viscoelasticity implying the assumptions of linearity and superposition, which means that the strain responses along the time to past histories of the stress can be added.

While the bases of this theory were set in theoretical continuum mechanics in the first years of 20th century by Volterra [36] under the definition of hereditary elasticity, it is only in the early forties that this theory was recognized by structural engineers as the proper mathematical tool for creep analysis of concrete structures. Pioneers of theses early attempts were Maslov in 1941 [32] and the school of Soviet engineers, under the leadership in particular of Gvozdev, all along the following more than three decades (see e.g. [3,26]). McHenry [31] in USA (1943) substantiated this trend through extended experimental confirmation, in creep tests of sealed specimens, of the principle of superposition inherent to Volterra's theory. He also stated the first two theorems of aging linear viscoelasticity for homogeneous concrete structures with rigid restraints later demonstrated by Levi (1948-1951) [29, 30]. Significant contributions in the application of ageing linear viscoelasticity to structural creep problems were later given, up to present times, by Bazant (see e.g. [4-9,14,15,28] and the list of downloadable papers from his website at Northwestern University), by the author, with the statement of the two other theorems of aging linear viscoelasticity for homogeneous concrete structures subjected to single or multiple changes of structural systems (see e.g. [16, 17, 20-23,35] and author's contributions to [2, 18,19,25]), and by other authors (see among others references [33-35]). For the group of theorems of aging linear viscoelasticity for homogeneous concrete structures with rigid restraints see here Section 4.2.2.b.

Although some significant deviations from linearity and the principle of superposition were detected by various authors along the past decades (see e.g. [6,9,27,37]), in particular in presence of variations of humidity and temperature with the corresponding changes in the rate of ageing (or hydration), unloading implying stress and strain reversals, cracking and other damage of concrete, bond slip, yielding or nonlinearity of reinforcement bonded to concrete, and of cyclic creep, the idea of basing time-dependent creep analyses of concrete structures on this principle, and the consequent theory of aging linear viscoelasticity, became generally accepted as a simplified assumption for design practice of current structures, when considering the average behaviour of massive beams and plates in which the water content and temperature cannot vary substantially or rapidly, and, especially, in the early stages of structural conception and design [1,2,4-6,16-25,28, 33-35]. Within these limits, the aging linear viscoelastic approach, first adopted in a pre-standard document by CEB in 1978 Model Code [18] and extensively applied in the related CEB Design Manual [22], is systematically adopted in recent guidance documents, recommendations and codes by international pre-standard and standard institutions [1,2,23-25].

More refined analyses taking into account the above mentioned deviations from superposition and the related linear aging viscoelastic solutions, with due account for the various causes of nonlinearity, are requested on the contrary in the final detailed reliability assessments, with particular regards to creep-sensitive, large and complex sequential structures (see the second option in Section 4.2.1).

The aging linear viscoelastic approach is based on the introduction of Volterra's hereditary integral equation as constitutive law for concrete in the following two equivalent forms, representing the generalization through the principle of linear superposition of the responses of concrete to a unit sustained uniaxial imposed stress or, respectively, strain, for time-variable values of these imposed actions after an initial finite step imposed at some age t_0:

$$\varepsilon_\sigma(t)=\sigma(t_0)J(t,t_0)+\int_{t_0}^{t} J(t,t')d\sigma(t') \tag{1}$$

$$\sigma(t)= \varepsilon_\sigma(t_0)R(t,t_0)+\int_{t_0}^{t} R(t,t')d\varepsilon_\sigma(t') \tag{2}$$

where $\varepsilon_\sigma(t)= \varepsilon(t) - \varepsilon_n(t)$ is the strain due to the stress $\sigma(t)$, $\varepsilon(t)$ being the total strain and $\varepsilon_n(t)$ the stress-independent strain (shrinkage and thermal strains), and $J(t,t')$ and $R(t,t')$ are the compliance and relaxation functions, representing, respectively, the strain or stress response at time t to a unit sustained constant uniaxial imposed stress ε_σ or strain *,t'* applied at time t' [2,4-6,21,23,25,28].

In equations (1) and (2) the hereditary integrals are written as Stieltjes integrals in order to admit discontinuous stress histories after the initial step at t_0. If $\sigma(t)$ and $\varepsilon_\sigma(t)$ are continuous after t_0, substitutions $d\sigma(t') = [d\sigma(t')/dt'] dt'$ and $d\varepsilon_\sigma(t') = [d\varepsilon_\sigma(t')/dt'] dt'$ yields the ordinary (Rieman) integrals[1].

The compliance and relaxation functions are reciprocally related by the integral equation:

$$1=R(t_0,t_0)\, J(t,t_0) \int_{t_0}^{t} J(t,t')dR(t',t_0) \tag{3}$$

where $dR(t',t_0)=\partial R(t',t_0)/dt'$.

[1] Some minor differences in the form of presentation of these fundamental equations currently exist in documents [2,23-25]. They will be eliminated adopting the formulations (1) to (3) of the present text and related definitions here above, in particular in the new editions of the fib Model Code and of Eurocode 2, both foreseen for 2020, and in the final edition of ACI 209.3R-XX, currently under revision by ACI TAC Technical Activity Committee.

4.2 Overview of current approaches for time-dependent structural analysis in codes and technical guidance documents

As anticipated in Section 3, in the most general case, the analysis on the basis of the theory of aging linear viscoelasticity of structures characterized by the presence of different concrete portions with a constitutive law of the type of Eq. (1) and, the case being, elastic (steel) portions, and subjected to complex histories of application of loads and imposed deformations and of changes of structural system, leads to a system of Volterra hereditary integral equations [2-4,6,17,20,22,23,25,26,28,34,35,37]. It is now well established that these equations, as well as Eq. (3) relating the compliance and relaxation functions, can be solved analytically only for some simple forms of the compliance function $J(t,t')$. Unfortunately, such forms, proposed in the past by many researchers (e.g. Macmillan, Glanville, Dischinger, Arutyunian, Levi, Ulickii, Prokopovich, Gvozdev, Aleksandrovskii,, Illstone, Nielsen, et. al) represent long-term creep and creep for high ages at loading very poorly [2,6,9,22,23,28]. One of the last proposals of this type was due to Rüsch and Jungwirth (1973) and was incorporated in CEB Model Code 1978 [18], and in CEB Design Manual [22] as one of the computational options.

These earlier forms have by now been abandoned since a numerical solution of Volterra integral equations can be easily obtained [4] for any form of the compliance function $J(t,t')$, as e.g. those provided by modern prediction models indicated in Section 3. Alternatively, numerical computations can be made more efficient converting the compliance function to an equivalent rate-type law, which allows also to capture the phenomena causing deviations from the principle of superposition in time.

These two very general approaches for the numerical solutions of creep analysis problems, which do not require the introduction of any unsatisfactory simplification for the creep compliance, are normally referred to as "General method". They are discussed here at Section 4.2.1.

On the other hand, when a highly refined analysis is not required, and in the conceptual and preliminary design stages, some convenient simplifications may be adopted, without the need, in this case too, to introduce any distorted form for the compliance. These simplifications are applied namely:

a) at the constitutive level, through the adoption of an approximate algebraic formulation for the constitutive law of Eq. (1); the very efficient algebraic formulation offered by the method known as Age Adjusted Effective Modulus (AAEM) method [2,5,9,22,23,25,28] is presented at Section 4.2.2.*a*;

b) at the level of the structural model, by the introduction of the approximate assumption of an effective rheological homogeneity for the concrete structure, or for the concrete part of a structure which includes steel structural elements, as discussed in Section 4.2.2.*b* [2,20,22-25,35].

Approach *b)* is particularly helpful at the conceptual design stage.

The general numerical method and the two alternative simplified approximate approaches *a)* and *b)* are the basic three options considered within the current internationally harmonized format for the evaluation of the structural response to the time-dependent behaviour of concrete adopted in most of recent pre-standard and standard guidance documents and recommendations [2,23-25]. They are briefly discussed in the sequel. For more information reference shall be made to Section 7.2.4 of *fib* Model Code 2010 [25] and Section 4.1.6 of *fib* Structural Concrete Textbook [23], as well as to ACI 209.3R-XX [2].

A common advantage of all these options, in the present scenery of a lack of consensus at the international level on the most appropriate creep prediction models for concrete to be adopted in the analysis of the structural reliability, is represented by the fact that they are not dependent from any specific prediction model formulation. On the contrary, they allow the exploration of the structural response for any creep prediction model considered appropriate in the specific case, as well as the comparison of results obtained incorporating in the computational process different prediction models.

4.2.1 General method

In this general approach, current computational algorithms for the elastic analysis of structures, like e.g. finite elements analysis procedures, are modified through the incorporation of the hereditary integral-type constitutive law of Eq. (1) formulated in an incremental form adapted to the numerical solution.

As anticipated here above, two distinct options are possible. They consist, respectively, in the recursive numerical procedure ensuing from replacing the history integrals over the past stress or strain histories with a sum [2,4,17,20,22,23,25,28,34] (*first option*), or, alternatively, in the transformation of the integral-type constitutive law into a rate-type creep law [2,9,15,25,28,37] (*second option*).

The *first option* has the advantage of incorporating directly, without the need of any adaptation, any of the present still numerous different formulations for the creep compliance $J(t,t')$ provided by prediction models proposed in the literature and in guidance documents at pre-standard and standard level, and characterizing the related integral-type constitutive Eq. (1). Convenient instructions for the recursive numerical treatment of the hereditary integral formulations are given in Section 7.2.4.11 of *fib* Model Code 2010 [25], and a detailed algorithm is provided in references [2,4,23,34].

In what concerns this option, it must be observed that in very large and complex structural systems, the storage of the entire stress or strain history at each integration point of each finite element and the evaluation of the history integrals can cause high computational loads. Although the ever increasing computational speeds and capacities of modern personal computers tend to overshadow this type of problem, the computation can be made more efficient adopting the second option, which is briefly discussed in the sequel, evidencing its advantages and open problems.

This *second option* consists in converting the integral-type constitutive creep law arising from the principle of superposition to an equivalent rate-type creep law with internal variables, whose current values account for the previous history of viscoelastic strain. In this case, the strain history needs not be stored because it is implied by the current values of these variables. The key property enabling the rate-type analysis is the fact that any realistic integral-type stress-strain relation of aging viscoelasticity can be approximated with any desired accuracy through appropriate procedures by a rate-type creep law visualized by a Kelvin chain model [2,9,15,28,37].

Rate-type laws are particularly helpful for the solution of structural problems by means of the finite elements method, because they are immediately compatible with this computational approach. They also make it easier to take into account the effects of numerous changes in the structural system, as in complex and sequential reinforced and/or prestressed concrete or composite steel and concrete constructions and spatial structures, like e.g segmental cantilever built or cable-stayed bridges, high-rise and super-tall buildings, etc.

As anticipated in Section 3, the rate-type form is also advantageous to capture, in addition, the phenomena indicated in Section 4.1 causing deviations from the principle of superposition in time, i.e. from the solutions based on aging linear viscoelasticity. In modelling and analysis of prestressed concrete structures, the rate-type form for the time-dependent constitutive law for concrete facilitates dealing also with another highly nonlinear phenomenon represented by the stress relaxation of steel tendons, which is not viscoelastic. The amount of steel relaxation may be significantly influenced by the strain variation in concrete and thus needs to be taken into account in the comprehensive time-dependent analysis of the structure.

All these phenomena cannot be taken into account through the hereditary memory integrals of the integral-type linear viscoelastic creep law (1) based on the assumptions of linearity and superposition in the time of creep effects, because cracking, humidity and temperature effects, unlike concrete creep, do not have a viscoelastic memory, and because cracking and steel relaxation are nonlinear phenomena. They make the stress-strain relation nonlinear and, upon their inclusion, the rate-type stress-strain relation ceases to obey the principle of superposition [2,9,15,37].

It may also be noted that for these more refined approaches local constitutive laws for concrete (creep and shrinkage laws for a material point) should be preferred, instead of through-thickness average constitutive laws [9,37]. With reference to the group of creep and shrinkage prediction models indicated in ACI 209.2R-08 Guide [1], only model B3 [7,37] offers such option.

The importance of the rate-type approach has recently been highlighted as well by the need to perform inverse analyses intended to interpret the time-histories, incorporated in the new RILEM data bank, of grossly excessive deflections of a number of segmental prestressed concrete bridges, for the purpose of contributing to the updating and validating of prediction models (with special regard to creep) for multi-decade predictions [14,15,37-40].

When the rate-type creep law is used, the structural creep problem can be reduced to a system of first-order ordinary differential equations in time with age-dependent coefficients. In order to avoid computational intricacies, it is more efficient to convert the incremental stress-strain relation for each time step to a quasi-elastic incremental stress-strain relation. Thus, the structural creep problem gets reduced to a sequence of elasticity problems with initial strains [2,9,15,28,37].

From an operational point of view, e.g. ACI 209.3R-XX [2] provides guidelines based on the most advanced debate and research outcomes for the computational procedures to be adopted for the transformation of the hereditary integral-type constitutive laws characterizing current creep prediction models into rate-type laws, and for the related numerical approach consisting in a sequence of elasticity problems with initial strains. Guidance is given also on the criteria to be adopted to account for all the above evidenced complex effects implying deviations from the principle of linear superposition. However, reference to the cited specialized literature is still essentially required for the moment being [9,15,37].

ACI Committee 209 *Creep and Shrinkage in Concrete* is consequently currently planning, through a specific Subcommittee (209-0D *Numerical Methods and 3D Analyses*), further committee activity intended to provide convenient practical guidelines for 3D time-dependent numerical analyses based on the rate-type creep approach for the reliability checks of complex spatial and sequential reinforced and/or prestressed concrete structures, composite steel and concrete constructions, and in general of all large structures of high creep sensitivity. These guidelines should enable practicing engineers to establish proper computational procedures and programs for construction-stage analyses and subsequent time-history analyses for the reliability checks in the long term, until the end of the specified service life, and/or to use for the same scopes, with adequate understanding and recognition of possible deficiencies, commercial software intended for these purposes.

4.2.2 Simplified approaches

a) AAEM method

The Age Adjusted Effective Modulus (AAEM) method is based on the adoption of the algebraic formulation (5) for the constitutive Eq. (1), when the compliance $J(t,t')$ is separated, as it is frequently done, into a nominal initial elastic strain and a creep strain according to the expression (4) introducing the creep coefficient $\varphi((t,t_0)$:

$$J(t,t_0)=\frac{1}{E_c(t_0)}+\frac{\phi(t,t_0)}{E_c(t_0)} \tag{4}$$

$$\varepsilon_\sigma(t)=\varepsilon(t)-\varepsilon_n(t)=\sigma(t_0)J(t,t_0)+\frac{[\sigma(t)-\sigma(t_0)]}{E_c(t_0)}[1+\chi(t,t_0)\phi(t,t_0)]=$$

$$=\frac{\sigma(t_0)}{E_{c,ef}(t,t_0)}+\frac{\sigma(t)\text{-}\sigma(t_0)}{E_{c,adj}(t,t_0)} \tag{5}$$

where $\chi(t,t_0)$ is the aging coefficient related to the compliance and relaxation functions by Eq. (6):

$$\chi(t,t_0)=\frac{1}{1\text{-}R(t,t_0)/E_c(t_0)}-\frac{1}{E_c(t_0)J(t,t_0)\text{-}1}=\frac{E_c(t_0)}{E_c(t_0)R(t,t_0)}-\frac{1}{\varphi(t,t_0)} \tag{6}$$

and $E_{c,ef}$ and $E_{c,adj}$ are the effective modulus and the age-adjusted effective modulus, respectively.

Eq. (5), with Eq. (6) defining the aging coefficient, represents the exact substitute of Eq. (1) for all problems characterized by strain and stress histories resulting from linear combinations of a creep and a relaxation problem, i.e. for strain histories characterized by an initial jump at time $t = t_0$ followed for $t > t_0$ by a strain variation linearly related to the compliance function $J(t,t_0)$ (or, equivalently, to the creep coefficient $\varphi(t,t_0)$), and consequent corresponding stress histories characterized by an initial jump followed by a stress variation linearly related to the relaxation function $R(t,t_0)$ [2,5,9,22,23,25,28].

This includes a broad range of strain and corresponding stress histories. With sufficient accuracy, its use may be extended to cover a large number of actual histories in structures showing an initial finite or zero value at $t = t_0$ and a time-dependent part varying at a gradually decreasing rate over wide time intervals. In current use of the AAEM method, Eq. (5) is given a quasi-elastic incremental formulation relating the variations of the strain and of the stress occurring after t0 .

If the strain and stress histories involve several subsequent sudden changes at times $t_i > t_0$, then the AAEM method must be applied separately for each increment and the results then superimposed; in other words the responses to multistep histories can be obtained by superimposing the solutions for several one-step histories.

As long-term values of $\chi(t,t_0)$ are contained in a rather narrow band for different values of t0, especially for GL2000 and B3 models, they are frequently approximated in the practice by a fixed value $\chi = 0.8$ when a high accuracy in the computations is not required, as e.g. in the conceptual and preliminary design stages and in the reliability assessment of structures of low sensitivity to time-dependent effects [2,23-25].

Originally formulated for homogeneous concrete structures, the method can be used by extension and with adequate accuracies also for non-homogeneous concrete structures, and structures incorporating steel elements [2,22,23,25,28].

It must be noted that in the creep prediction model of fib Model Code 2010 [25] (as well as in GL2000 model [1]) a different creep coefficient $\varphi_{28}(t,t_0)$ (representing the ratio between the creep strain and the nominal initial elastic strain at 28 days) is introduced in Eq. (4). Therefore, equations (5) and (6) must be adapted accordingly (see Section 7.2.4.10 of *fib* Model Code 2010).

b) Aging linear viscoelastic solutions for effective homogeneous concrete structures of averaged rheological properties

This approach is based on the introduction of the approximate assumption of an effective rheological homogeneity for the entire concrete structure, or for the concrete part of a structure which includes steel structural elements.

In fact it may be considered that, if we exclude the cases where important differences in creep properties are present, due e.g. to large differences in size of concrete structural elements, or in their age (as is typical in sequential constructions that can be smeared along many months, as e.g. in supertall buildings), the influence of creep differences is frequently significantly outshined by the large creep strains developing at long term. Thus these differences can sometimes be neglected, in particular in the conceptual and early stages of design, when estimating the general trends of the long-term response of the structure and assessing its reliability with the degree of accuracy which is appropriate for these stages, or when checking the general trends of the outcomes of detailed numerical calculations, and whenever a high computational accuracy is not requested with respect to the importance of the structure and its sensitivity to long-term effects.

This approach, which makes use of the simple solutions provided by the fundamental theorems of aging linear viscoelasticity for homogeneous concrete structures with rigid restraints, and of some compact formulations for homogeneous concrete structures with additional steel structural elements, benefits of the concise and conceptually clear character of all these solutions [2,20,22-25,29,30,35].

For the detail of these solutions, reference can be made in particular to Sections 7.2.4.8-9 of *fib* Model Code 2010 [25] and Sections 4.1.6 (8)-(9) of *fib* Structural Concrete Textbook [23], as well as to ACI 209.3R-XX [2] and the referenced literature therein.

In the case of homogeneous concrete structure with rigid restraints, the first two theorems of aging linear viscoelasticity state the lack of influence of creep along the time on the elastically evaluated state of the stresses or, respectively, of the strains under imposed (constant or variable) loads or, respectively, deformations. Correspondingly, they state the fact that the variations of the consequent induced deformations or, respectively, stresses are modelled through the compliance function $J(t,t_0)$ or, respectively, the relaxation function $R(t,t_0)$. See equations (7.2-27) to (7.2-34) of *fib* Model Code 2010 [25].

Two further theorems demonstrated by the author [2,20,22-25,35] define the effects, in a structure loaded by a system of constant loads applied at time t_0, of single or multiple changes of structural system (resulting by the introduction of additional rigid restraints) applied, respectively, at time $t_1 \geq t_0^+$, or at subsequent times $t_i \geq t_0^+$ ($i = 1,...,j$). These effects, in terms of redistribution of the system of internal stresses and external reactions, are modelled by the redistribution function $\xi(t,t_0,t_i)$ measuring, at time t, the creep induced partial acquisition by the structure, after the modification of its structural system at times t_i, of the difference between the elastic stress distribution characterizing the structural system modified at time t_i and the elastic stress distribution previous to this modification. See equations (7.2-35) and (7.2-37) of *fib* Model Code 2010 [25].

The redistribution function is related to the compliance or to the relaxation functions $J(t, t')$ and $R(t, t')$, respectively, by the following integral expressions:

$$J(t,t_0) - J(t_i,t_0) = \int_{t_i}^{t} J(t,t')\,d\xi(t',t_0,t_i) \qquad (7)$$

$$\xi(t,t_0,t_i) = \int_{t_i}^{t} R(t,t')\,dJ(t',t_0) \qquad (8)$$

Eq. (7), corresponding to Eq. (7.2-36) of *fib* Model Code 2010 (for $t_i = t_1$), is normally adopted for the determination of the redistribution function $\xi(t,t_0,t_i)$ from the given compliance function $J(t, t')$, which requires the solution of a Volterra integral equation. The recursive numerical procedure indicated in Section 4.2.1

(*first option*) may be adopted.

Some compact formulations can also be obtained for concrete structures with additional steel structural elements, in the cases where the structure can be schematically represented by an homogeneous concrete structure subjected to a system of sustained constant loads applied all at the same time, and restrained by a system of n redundant elastic restraints also introduced all at the same time, either before or immediately after the loads; the condition of constant imposed deformations at the points of applications of the redundant elastic restraints may be considered as well (see [2, 23, 25], and the literature referenced in these documents).

These solutions, which are formulated in matrix form, represent the extension to the case of elastic restraints of the aging linear viscoelastic solutions for homogeneous structures with rigid restraints discussed here above. They are based on some fundamental functions (called reduced relaxation functions and denoted R_i*), representing structural variables that depend both on the creep compliance $J(t, t')$ of the concrete structure and on the eigenvalues ω_i of the elastic coupling matrixes between the two parts (the concrete structure and the system of the n redundant elastic restraints, with $i = 1,2,\ldots$n denoting the generic elastic restraint). For a given creep prediction model, and for the corresponding compliance J, the determination of the n reduced relaxation functions R_i* requires the solution of the n Volterra integral equations linking the reduced relaxation functions R_i* to appropriately modified compliance functions J_i*depending on the eigenvalues ω_i.

In the case of constant imposed loads with the n redundant elastic restraints introduced before the loads, differently from the case of homogeneous concrete structures with rigid restraints, for which the invariance of the state of stress is demonstrated by the first theorem of linear viscoelasticity (see above), these compact formulations show that the initial elastic state of stress in the structure and in the restraints is significantly altered by creep. The higher the deformability of the elastic restraints, the higher is the difference between the initial and long-term values of the state of stress. In the long term, the system of stresses tends to approach the solution corresponding to the case of rigid restraints.

A reduction of the time dependence of the state of stress under permanent loads when highly deformable elastic restraints are adopted can be obtained through a convenient stressing of these restraints. In fact, in the theoretical case of an effective homogeneous structure with elastic restraints introduced all at the same time at the time of loading, the invariance of the stress state is obtained forcing the restraints up to the values of the rigid restraints reactions.

In the case of modification of the restraint conditions after loading through the introduction of additional elastic restraints immediately after loading, the theoretical solutions show that the system of elastic restraints contributes to a lower degree to the variation of the original system of the stresses in the structure, attracting lower values of restraint reactions, with respect to the case of delayed additional rigid restraints discussed here above.

These compact formulations for concrete structures with additional elastic (steel) structural elements can be adopted e.g. in the preliminary conceptual design of tied concrete arches and frames, or cable stayed concrete bridges and structures, while ignoring at this stage of the design the heterogeneities of concrete and the sequential construction approach normally adopted in the practice, which shall be properly considered, on the contrary, in the final construction-stage analyses and subsequent time-history analyses.

4.3 Design aids

When, as it is generally the case, reference is made as primary function to the compliance function $J(t, t')$ of one of the recommended creep prediction models, the corresponding secondary structural variables represented by the relaxation and redistribution functions $R(t, t')$ and $\xi(t,t_0,t_i)$, can be evaluated, as indicated in the preceding Sections through the solutions, respectively, of the Volterra integral Eqs. (3) and (7), adopting the recursive numerical procedure indicated at Section 4.2.1 (*first option*).

Different values of t_0 must be considered and repeated solutions of Eq. (3) must be performed to obtain $R(t, t')$ from $J(t, t')$, while different values of t_0 and ti must be considered and repeated solutions of Eq. (7) must be performed to obtain $\xi(t,t_0,t_i)$. The aging coefficient $\chi(t,t')$, after the determination of the relaxation function $R(t, t')$, can be determined from Eq. (6) for different values t' of t_0. Design aids can be easily provided for this scope [35,41]. The program *creep* (which will be possible to freely download from the website *creepanalysis* [41] currently under revision) allows an immediate numerical determination of $R(t, t')$, $\xi(t,t_0,t_i)$, $\chi(t,t')$ from the primary function $J(t, t')$.

A comparison of the trends with respect to time and age of the diagrams of the compliance, relaxation and redistribution functions, and of the ageing coefficient obtained for three of the prediction models of ACI.209 2R [1] is shown in [2,21,23]. Comparisons with the trends resulting from the *fib* Model Code 2010 prediction model [25] and the model B4 proposed by Bazant and his research group [14,15,38-40] are currently under way.

5. Conclusions

Modern concrete structures, realized through complex sequential construction techniques and/or characterized by significant non-homogeneities, are in general very sensitive to the effects of the delayed deformations exhibited by the time-dependent behaviour of concrete (creep and shrinkage).

Current internationally agreed and harmonized formats and procedures for the analysis of these structural effects adopted in recent guidance documents, recommendations and codes have been illustrated in this paper. This harmonization process represents a valuable result compensating in part the persisting divergences and uncertainties in what concerns the establishment of reliable prediction models for the time-dependent behaviour of concrete, with special regards to creep and its long-term trends. This significant result was obtained through intense debate and cooperation between different schools and scholars at worldwide level, inspired and promoted along the last four decades by international pre-standard and standard institutions in the field of structural concrete, with a coordination action by the author.

In what concerns the accurate numerical computational procedures of the general method, some specific guidance criteria are still required for an easy and user friendly implementation of the rate-type approach, intended to be at the same time coherent with the most advanced research outcomes. In fact, the rate-type approach is more advantageous and computationally efficient, with respect to the integral approach, in the detailed evaluation of the effects of the time dependent behaviour of concrete in the construction-stage and long-term reliability analyses of large complex, sequential and highly creep sensitive structures modelled by FEM. It allows also in particular to take into account the influence of various inelastic and nonlinear phenomena which are responsible of causing deviations from aging linear viscoelasticity, such as tensile cracking, cyclic creep, and stress relaxation in prestressing tendons at variable strain, as well as the effects of humidity and temperature variations.

ACI Committee 209 *Creep and Shrinkage in Concrete*, will contribute through a specific subcommittee to the development of practical guidance rules for this scope.

Acknowledgments

The publication in 2013 within the new fib Model Code for Concrete Structures 2010 [25] of Section 7.2.4 *Analysis of structural effects of time-dependent behaviour of concrete* marks the successful conclusion of the long process of international harmonization at worldwide pre-standard level of guidance criteria in this specific domain. The author is particularly indebted to the late CEB Honorary Member and member of the Academy of Construction and Architecture of USSR Alexei A. Gvozdev, for long-time head of the research laboratory of NIIZhB, the Institute for Concrete and Reinforced Concrete now named after him, for his crucial role in encouraging and assisting the author, in the 1960-70s, in the initial steps of transporting into the international pre-standard ambient of CEB-FIP (the parental organizations of *fib*) the new advanced format for creep-analysis based on the theory of ageing linear viscoelasticity, to which the school of Soviet scientists and Gvozdev himself had given important contributions. This paper must be intended as a homage to his memory.

The author pays homage also to the memory of his own Italian maestro Franco Levi, for long time President of CEB, and subsequently of FIP, for his first incentive in the establishment of a CEB Design Manual on Structural Effects of Time-dependent Behaviour of Concrete [22]. The contribution of S.V. Alessandrosky, Z.P. Bazant, J. Fauchart and D. Jungwirth in the first drafting of the Manual, and of P. Napoli, F. Mola and M. Koprna in the final editing, is acknowledged. The author is also indebted to D. Carreira, W. Dilger and M. Sassone, members of the editorial team for the current extended draft of ACI 209.3R-XX guidance document *Analysis of Creep and Shrinkage Effects in Concrete Structures* [2], and to Z.P Bazant for his contribution to the discussion of the document and to the drafting of the text on the rate-type approach.

Finally, the author is indebted to all scholars and researchers that contributed to the debate on the analysis of structural effects of time-dependent behaviour of concrete along the four past decades within CEB, *fib* and ACI ambient, and to all his co-workers of the *Creepanalysis* research group of the Politecnico di Torino, in particular M. Sassone and C. Casalegno.

References

1. ACI 209.2R-08, *Guide for Modeling and Calculation of Shrinkage and Creep in Hardened Concrete,* American Concrete Institute, Farmington Hills, MI, 2008, 48 p.

2. ACI 209.3R-XX, *Analysis of Creep and Shrinkage Effects on Concrete Structures,* Chiorino M.A. (Chairm. of Edit. Team), ACI Committee 209, March 2011, Final approved draft 2013; currently under revision by ACI TAC Technical Activity Committee, 228 p.

3. Aleksandrovskii S. V., *Analysis of Plain and Reinforced Concrete Structures for Temperature and Moisture Effects (with Account of Creep)* (in Russian), Stroyizdat, Moscow, 1966, 443 p.

4. Bazant Z.P., Numerical Determination of Long-range Stress History from Strain History in Concrete, *Material and Structures,* Vol. 5, 1972, pp. 135-141.

5. Bazant Z.P., Prediction of Concrete Creep Effects Using Age-adjusted Effective Modulus Method, *Journal of the American Concrete Institute,* V. 69, 1972, pp. 212-217.

6. Bazant Z. P., Theory of Creep and Shrinkage in Concrete Structures: a Précis of Recent Developments, *Mechanics Today,* vol.2, Pergamon Press, New York, 1975, pp. 1-93. See also: RILEM TC-69, Material Models for Structural Creep Analysis (principal author Z.P. Bazant), Chapter 2 in *Mathematical Modeling of Creep and Shrinkage of Concrete,* Z.P. Bazant, ed., J. Wiley, Chichester and New York, 1988, pp. 99-215;

RILEM TC-69, Creep Analysis of Structures (principal authors Z.P. Bazant and O. Buyukozturk), Chapter 3, ibid. pp.217-273.

7. Bazant Z. P., and Baweja S., Creep and Shrinkage Prediction Model for Analysis and Design of Concrete Structures - Model B3, in: A. Al-Manaseer ed., *The A. Neville Symposium: Creep and Shrinkage - Structural Design Effects,* ACI SP-194, American Concrete Institute, Farmington Hills, Michigan, 2000, pp. 1-83.

8. Bazant Z.P., Hubler M.H., Yu Q., Pervasiveness of Excessive Segmental Bridge Deflections: Wake-Up Call for Creep, *ACI Structural Journal,* Vol. 108, No. 6, Nov.-Dec. 2011, pp. 766-774.

9. Bazant Z. P., Li G.-H., and Yu Q., Prediction of Creep and Shrinkage and their Effects in Concrete Structures: Critical Appraisal, Proc., *8th Int. Conf. on Creep, Shrinkage and Durability of Concrete and Concrete Structures - CONCREEP 8,* Vol. 2, T. Tanabe, et al. eds., CRC Press, Boca Raton, FL, 2009, pp. 1275–1289.

10. Bazant Z.P., and Li G.-H., Unbiased Statistical Comparison of Creep and Shrinkage Prediction Models, *ACI Materials Journal*, Vol. 105, No. 6, Nov.-Dec. 2008, pp. 610-621.

11. Bazant Z.P., and Li G.-H., Comprehensive Database on Concrete Creep and Shrinkage, *ACI Materials Journal* Vol. 105, No. 6, Nov.-Dec. 2008, pp. 635-638.

12. Bazant Z. P., and Prasannan S., Solidification Theory for Concrete Creep: I. Formulation, *Journal Eng. Mech.,* 115(8), 1989, pp. 1691–1703.

13. Bazant Z. P., and Prasannan S., Solidification Theory for Concrete Creep: II. Verification and Application, *Journal Eng. Mech.,* 115(8), 1989, pp. 1704–1725.

14. Bazant Z. P., Yu Q., and Li G.-H., Excessive Long-Time Deflections of Prestressed Box Girders. I: Record-Span Bridge in Palau and Other Paradigms, *ASCE Journal. of Structural Engineering,* Vol. 138, No. 6, June 2012, pp. 676-686.

15. Bazant Z. P., Yu Q., and Li G.-H., Excessive Long-Time Deflections of Prestressed Box Girders. II: Numerical Analysis and Lessons Learned, *ASCE Journal of Structural Engineering,* Vol. 138, No. 6, June 2012, pp. 687–696.

16. Casalegno C., Sassone M., Chiorino M. A., Time-dependent Effects in Cable-stayed Bridges Built by Segmental Construction, *Proc. of Third International fib Congress incorporating the PCI Annual Convention and Bridge Conference.* Washington D. C., 2010, pp. 539-554.

17. Casalegno C., Sassone M., Chiorino M. A., Time-dependent Effects in Concrete Structures: a General Computational Approach, *Proc. of Structural Engineers World Congress SEWC 2011,* Como, Italy, (CD).

18. CEB, Comité Eurointernational du Béton and Fédération *Internationale de la Précontrainte, International System of Unified Standard Codes of Practice for Structures, Vol. II, CEB-FIP Model Code for Concrete Structures,* CEB Bulletin d'Information N° 124/125-E-F, 1978, 348 p.

19. CEB, *CEB-FIP Model Code 1990*, CEB Bulletin d'Information No. 213/214, Comité Euro-International du Béton, Lausanne, Switzerland, 1993, 437 p.

20. Chiorino M.A., A Rational Approach to the Analysis of Creep Structural Effects, in J. Gardner & J. Weiss (eds). *Shrinkage and Creep of Concrete,* ACI SP-227, 2005, pp.107-141.

21. Chiorino M.A. and Carreira D. J., Factors Affecting Shrinkage and Creep of Hardened Concrete and Guide for Modelling - A State-of-the-art Report on International Recommendations and Scientific Debate, *The Indian Concrete Journal,* Vol. 86, No. 12, December 2012, pp. 11-24. Errata, Vol. 87, No. 8, August 2013, p.33. Chiorino M.A. and Casalegno C.,, Evaluation of the Structural Response to the Time-dependent

Behaviour of Concrete: Part 1 - An Internationally Harmonized Format, Ibidem, pp. 25-36.

22. Chiorino M.A. (Chairm. of Edit. Team), Napoli P., Mola F., and Koprna M., *CEB Design Manual on Structural Effects of Time-dependent Behaviour of Concrete,* CEB Bulletin d'Information N° 142-142 Bis, Georgi Publishing Co., Saint-Saphorin, Switzerland, March 1984, 391 p.

23. Chiorino M.A. and Sassone M., Further Considerations and Updates on Time-dependent Analysis of *Concrete Structures, in Structural Concrete - Textbook on behaviour, design and performance,* 2nd edition, Vol. 2, Section 4.16, fib Bulletin 52, International Federation for Structural Concrete, Lausanne 2010, pp. 43-69.

24. EN 1992-2 *Eurocode 2: Design of Concrete Structures - Part 2: Concrete Bridges, Design and Detailing Rules,* Appendix KK, *Structural effects of time dependent behaviour of concrete,* 2005, pp. 63-67.

25. *fib, Model Code for Concrete Structures 2010,* Ernst & Sohn, 2013, 402 p.; see also Müller H. S. (Convener), *Code-type models for structural behaviour of concrete: Background of the constitutive relations and material models in the fib Model Code for Concrete Structures 2010,* State-of-art report, *fib* Bulletin No. 70, 196 p., November 2013.

26. Gvozdev A. A., Creep of Concrete (in Russian), *Proc. of the 2nd National Conference on Theoretical and Applied Mechanics. Mechanics of Solids, Mekhanika Tverdogo Tela,* Acad. of Sciences USSR, 1966, Moscow, pp. 137-152. (French translation: Le Fluage du Béton, *CEB Bulletin N° 64,* 1967).

27. Gvozdev A. A., Galustov K. Z., and Yashin A. V., On some deviations from the superposition principle in creep theory (in Russian), *Beton i Zhelezobeton,* 13(8), 1967,

28. Jirásek M. & Bazant Z.P., *Inelastic Analysis of Structures,* Wiley and Sons, 2002, 734 p.

29. Levi F., Sugli Effetti Statici dei Fenomeni Viscosi (On the Structural Effects of Viscous Phenomena, in Italian), *Rendiconti Accademia Nazionale dei Lincei,* Serie VIII, Vol. IV, fasc.3, pp. 306-311, fasc. 4, 1948, pp. 424-427.

30. Levi, F. and Pizzetti G., *Fluage, Plasticité, Précontrainte,* Dunod, Paris, 1951, 463 p.

31. McHenry D., A New Aspect in Creep of Concrete and its Application to Design, *Proc. ASTM,* Vol. 43, 1943, pp. 1069-86.

32. Maslov G. N., Thermal Stress States in Concrete Masses, with Account of Concrete Creep (in Russian), *Izvestia* NIIG, 28, 1941, pp.175-188.

33. Salençon J., *Viscoélasticité pour le Calcul des Structures,* Les Éditions de l'École Polytechnique, Les Presses des Ponts et Chaussées, Paris, 2009.

34. Sassone, M. and Casalegno, C., Evaluation of the Structural response to the Time-dependent Behaviour of Concrete: Part 2 - A General Computational Approach, *The Indian Concrete Journal,* Vol. 86, No. 12, December 2012, pp. 39-51. Errata, Vol. 87, No. 8, August 2013, p. 33.

35. Sassone M. and Chiorino M. A., Design Aids for the Evaluation of Creep Induced Structural Effects, in J. Gardner & J. Weiss (eds). *Shrinkage and Creep of Concrete,* ACI SP-227, 2005, pp. 239-259.

36. Volterra V., Sulle Equazioni Integro-Differenziali della Teoria della Elasticità (Integral-Differential Equations of the Theory of Elasticity, in Italian), *Rendiconti Accademia Nazionale dei Lincei,* Vol. XVIII, 2° Sem., 1909, pp. 295-301. See also: Volterra V., Sur les Equations Integro-Differentielles et leurs Applications, *Acta Mathematica,* G. Mittag-Leffler Ed., Stockholm, 1912, pp. 295-350 ; Volterra V., *Leçons sur les Fonctions de Lignes,* Gauthier-Villars, Paris, 1913, 230 p.

37. Yu Q., Bazant, Z.P. and Wendner R., Improved Algorithm for Efficient and Realistic Creep Analysis of Large Creep-Sensitive Concrete Structures, *ACI Structural Journal,* Vol. 109, No. 5, Sept-Oct. 2012, pp. 665-675.

38. Wendner, R., Hubler, MH, and Bazant, ZP., Optimization Method, Choice of Form and Uncertainty Quantification of Model B4 Using Laboratory and Multi-Decade Bridge Databases, *Material and Structures,* Vol. 48(4), 2015, pp. 771-796.

39. Wendner, R., Hubler, M.H., and Bazant,Z.P. (2015). Statistical Justification of Model B4 for Multi-decade Concrete Creep Using Laboratory and Bridge Data Bases and Comparisons to other Models, *Material and Structures,* Vol. 48(4), 2015, pp. 815-833.

40. RILEM Technical Committee TC-242-MDC (Z.P.Bazant, chair), RILEM draft recommendation: TC-242-MDC Multi-decade Creep and Shrinkage of Concrete: Material Model and Structural Analysis. Model B4 for Creep, Drying Shrinkage and Autogenous Shrinkage of Normal and High-Strength Concretes with Multi-Decade Applicability, *Material and Structures,* Vol. 48(4), 2015, pp. 753-770.

41. Creepanalysis, www.polito.it/creepanalysis, DISEG Dipartimento di Ingegneria Strutturale e Geotecnica, Politecnico di Torino, (currently under revision; reference should be made to new version to be edited in 2018).

Author:

Mario Alberto CHIORINO, Professor Emeritus of Structural Analysis, Polytechnic of Turin; National Member Turin Academy of Sciences; Honorary Member American Concrete Institute. Past Chair of ACI Committee 209 Creep and Shrinkage in Concrete.

Politecnico di Torino DAD, Viale Mattioli 39, I-10125.Turin, Italy.

e-mail: mario.chiorino@polito.it